parison vector and measurement vector. The angle associated with the comparison vector is the wanted bearing.

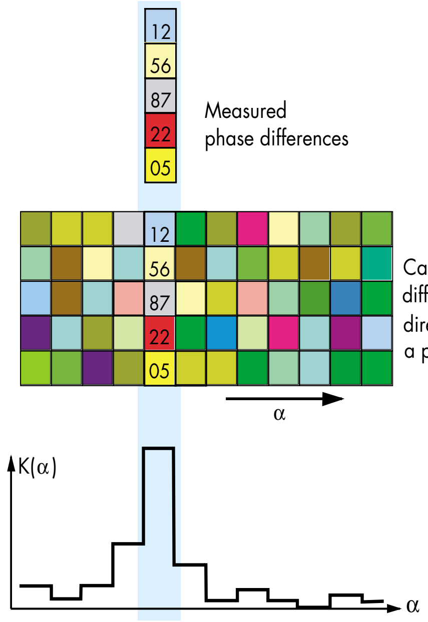

Fig. 17: Principle of correlation evaluation

This method is a special form of a beamforming algorithm [11], which is described in detail in the following section on direction finding using sensor array processing.

4 Direction Finding Using Sensor Array Processing

4.1 General

The development of the classical DF methods was aimed at designing antenna configurations and circuits that allowed bearing determinations to be as simple as possible. It was important to establish a simple mathematical relationship between the antenna signals and the direction of wave incidence largely independent of frequency, polarization and environment.

With the development of digital signal processing new approaches have now become possible:

- With high-speed signal processing chips being available, the requirement for a simple and frequency-independent relationship between the antenna signals and the bearing no longer applies. Even highly complex mathematical relationships can be evaluated in a reasonably short time for determining the bearing or handled fast and economically with the aid of search routines
- Numeric methods allow the separation of several waves arriving from different directions even with limited antenna apertures (high-resolution method, super resolution, multiwave resolution)

4.2 Basic design

A typical hardware configuration of a DSP-based direction finder is shown below.

The outputs of the individual antenna elements are usually taken first to a network which contains for instance

- test signal inputs and
- multiplexers if the number N of antenna outputs to be measured is greater than the number H of the receiver sections (tuner and A/D converter).

The signals are then converted to an intermediate frequency that is appropriate for the selected sampling rate of the A/D converter and digitized. To reduce the volume of data, the digital data are down-converted into the baseband. The complex samples of the baseband signal $x_i(t)$ $(i=1, 2, \dots N)$ are filtered for the desired evaluation bandwidth and applied to the bearing evaluation section.

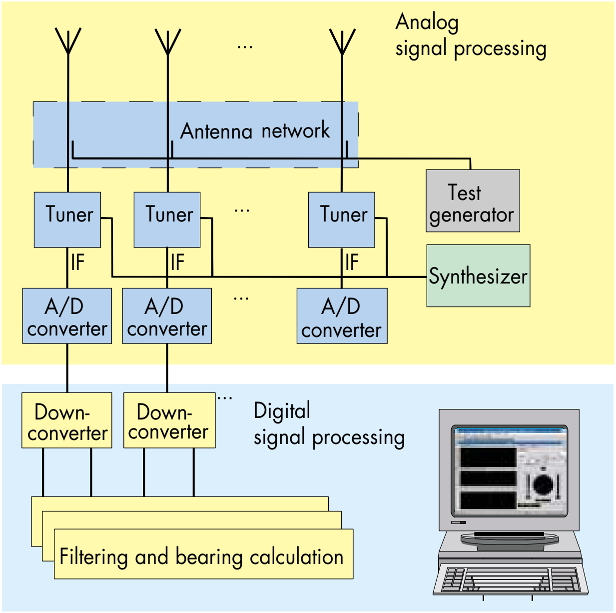

Fig. 18: Typical configuration of a DSP-based direction finder

4.3 Beamforming methods[1]

If analogously to the conventional antenna arrays the element signals x_i are multiplied by complex weighting factors w_i and added (Fig. 19), a sum signal is obtained which according to the resulting directional characteristic depends on the direction of wave incidence.

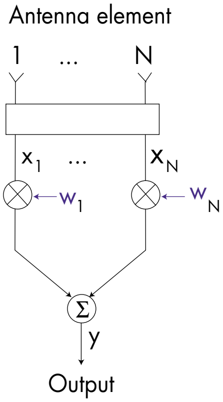

Fig. 19: Beamforming by weighting the outputs of an antenna array

The response of the output signal y to the change in the weighting factors w_i cannot be used for direction finding the same as with the classical rotating or goniometer direction finder. The difference is that with numeric beamforming the DF speed is only limited by the computing speed.

With **conventional beamforming algorithms** the phases of the weighting factors are chosen so that the weighted element signals are added in phase and thus yield a maximum sum signal if the wave arrives from the given direction α_r.

Fig. 20 shows the sequence of the direction finding process.

Fig. 21 shows the response of a linear array with five elements spaced 0.45λ apart to the variation of the direction with a wave arriving at an angle of 60°.

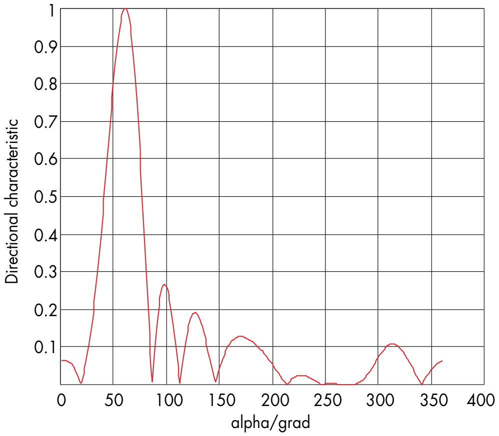

Fig. 21: Response of linear 5-element array to variation of direction (elements have cardioid directional characteristic)

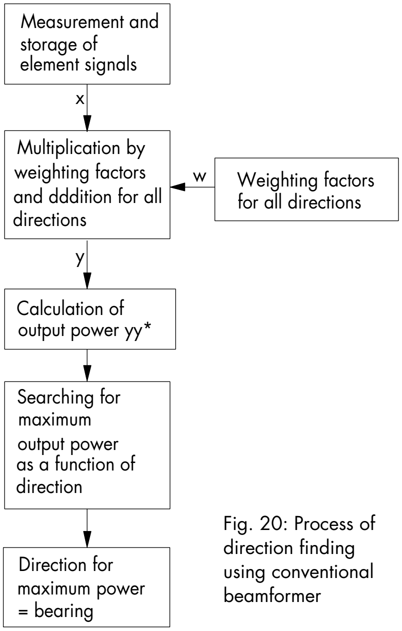

Fig. 20: Process of direction finding using conventional beamformer

If antenna arrays with largely the same elements and an array geometry describable by analytical means are used, the weighting factors can in the most cases directly be calculated from the array geometry.

If multiport antennas are used (Fig. 22, see next page), the port voltages u_i are as a rule measured as a function of the wave angle. The weighting is then given by

$$w_i(\alpha) = \frac{u_i^*(\alpha)}{u_0(\alpha)} = x_i^*(\alpha)$$

where u_0 is a suitable reference voltage and (.)* means complex conjugate. Since beamforming using general multiport antennas often does not produce a distinct directivity of the (synthetic) antenna diagram, the terms

- correlation method and
- vector matching

are used in this case too.

This is explained by the following: if the N measured complex antenna voltages are considered to be the coordinates of a vector in an N-dimensional space (measurement vector) and the N weighting factors the components of the weighting factor in the same space, beamforming is equivalent to the forming of a scalar product between these two vectors. If a normalization to the absolute values is made, the scalar product corresponds to the direction cosine between the two vectors. The direction cosine reaches its maximum if the directions agree (the vectors are matched); the Euclidean distance reaches its minimum.

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[1] The beamforming, correlation and Fourier methods are equivalent in their system theory: they all use generalized FIR filters with wave incidence angles as "spatial frequencies"

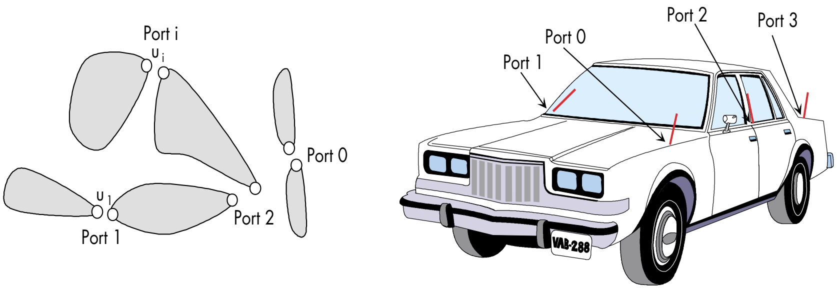

Fig. 22: General multiport antenna with application example

The direction cosine between two N-dimensional vectors corresponds to the correlation between two data sets with N elements. The distance square corresponds to the mean square error between two data sets to be matched.

4.4 High-resolution DF methods (super resolution)

If in the frequency channel of interest unwanted waves are received in addition to the wanted wave, conventional beamforming may lead to bearing errors as a function of the antenna geometry. There are two approaches to solve this problem:

- If the power of the interference wave component is smaller than that of the wanted wave component, the direction finder can be designed to minimize the bearing errors especially by choosing a sufficiently wide antenna aperture (see 5.1)
- If the interference wave component is greater or equal to the wanted wave component, the interference waves too have to be determined in order to be able to eliminate them. When using conventional beamforming algorithms, this means that the secondary maxima in the DF function have to be evaluated too. The limits are reached
  - if the ratio between primary maximum and secondary maxima of the directional characteristic becomes too small or
  - the angle difference between wanted and interference wave is smaller than the width of the main lobe

By optimizing the weighting factors, the level of the secondary maxima can be reduced but at the same time the width of the primary maximum is increased. The aim of the super-resolution (SR) DF method is to resolve this problem.

Minimum-signal direction finders are so to speak the grandfathers of the SR direction finders. In the early days of direction finding, the bearing of co-channel signals was taken by alternately suppressing the waves involved with the aid of a rotating loop. It is noteworthy that signals are separated by the acoustic monitoring of the modulation. To determine the loop null a correlation process with acoustic patterns is therefore required.

Adaptive antennas are antenna arrays with beamformers allowing automatic spatial suppression of interference waves [13], [14], [15]. In communication systems, optimization of the signal-to-noise ratio is the primary aim; in radio direction finding the weighting determined for signal suppression is used to determine the directions of wave incidence.

To this end the weighting of the beamformer is selected so that under certain auxiliary conditions the output power is minimized. In the case of the Capon beamformer [22] the auxiliary condition for setting the weighting is defined with the antenna gain remaining constant for a given direction α_r. If the incident waves are uncorrelated, the beamformer is adjusted for nulls to occur in all signal directions except for the direction α_r (Fig. 23).

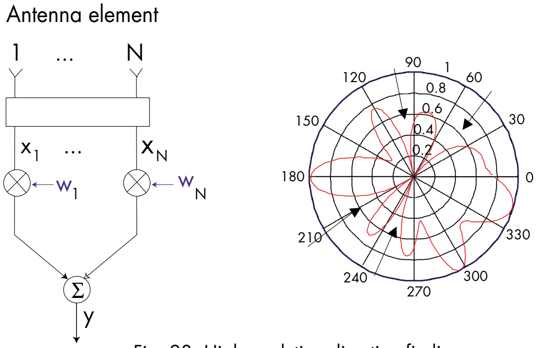

Fig. 23: High-resolution direction finding through nulling

If the direction of an incident wave coincides with the given direction α_r, there is a distinct maximum in the output power. Fig. 24 shows an example of the angular spectrum of a Capon beamformer with a 9-element circular array ($D/\lambda=1.4$) and five uncorrelated waves [23].

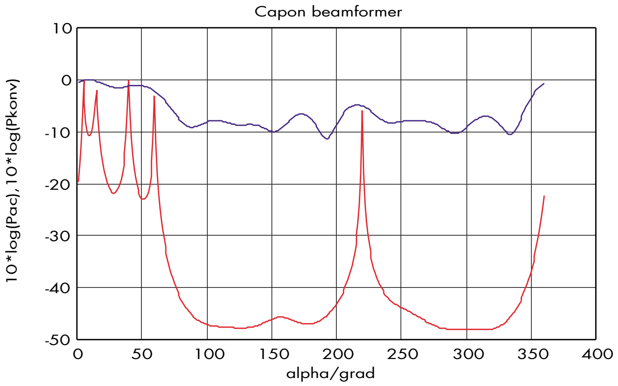

Fig. 24: DF function of Capon beamformers compared to conventional beamformer (S/N=100). Wave angle: 5°, 15°, 40°, 60°, 220°

Similar to a minimum-signal direction finder, the resolution strongly depends on the signal-to-noise ratio. Fig. 25 shows the same receiving scenario with noise increased by a factor of 10. The resolution of waves arriving at an angle of 5°and 10° is no longer possible.

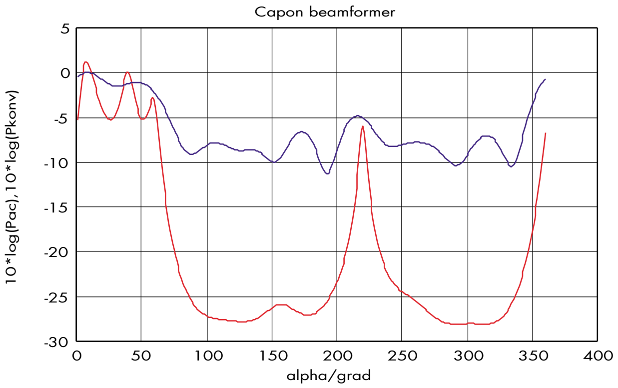

Fig. 25: DF function of Capon beamformer compared to conventional beamformer (S/N=10)

The socalled **subspace methods** are aimed at eliminating the effect of noise. This can be done by splitting up the N-dimensional space spanned by the element outputs into subspaces. The well-known MUSIC algorithm (**Mu**ltiple **Si**gnal **C**lassification) uses the fact that the signals lie perpendicular to the noise subspace. If the direction vectors are now projected to the noise subspace, nulls are obtained in the presence of signals that are independent of the noise level [16], [17], [24]. The reciprocal value is usually used as the DF function so that distinct peaks occur in the signal directions (Fig. 26).

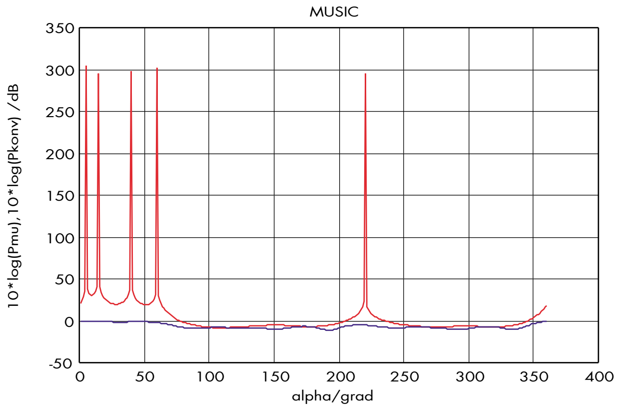

Fig. 26: DF function when using MUSIC algorithm (S/N=10)